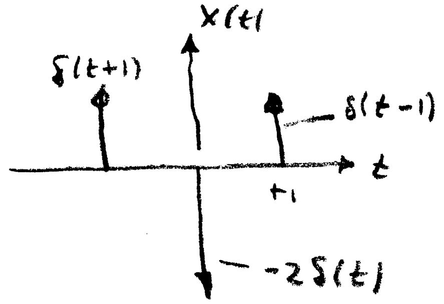
Homework Solutions

S8) i) $x(t) = \delta(t+1) - 2\delta(t) + \delta(t-1)$

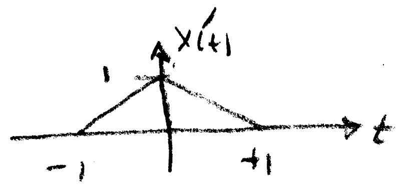


$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t+1) e^{-j\omega t} dt - 2\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt + \int_{-\infty}^{\infty} \delta(t-1) e^{-j\omega t} dt$$

$$= e^{-j\omega} - 2 + e^{+j\omega} = 2(\cos(\omega) - 1)$$

ii) Problem S7 developed the Fourier transform for

$$x'(t) = \begin{cases} t+1 & -1 < t < 0 \\ 1-t & 0 < t < 1 \\ 0 & |t| > 1 \end{cases}$$

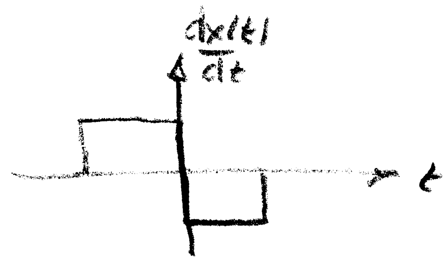


and obtained

$$X'(j\omega) = \frac{2}{\omega^2}[1 - \cos(\omega)]$$

Now

$$\frac{dx'(t)}{dt} = \begin{cases} 1 & -1 < t < 0 \\ -1 & 0 < t < 1 \\ 0 & |t| > 1 \end{cases}$$



and

$$\frac{d^2 x'(t)}{dt^2} = \delta(t+1) - 2\delta(t) + \delta(t-1)$$

So the second derivative of $x''(t)$, from 5.7 is equal to $x(t)$ for 58 i) above.

Applying the derivative rule for Fourier transforms to $x'(t)$ yields the following F.T. pairs.

$$x'(t) \xleftrightarrow{\mathcal{F}} \frac{2}{\omega^2}[1-\cos(\omega)] = X'(j\omega)$$

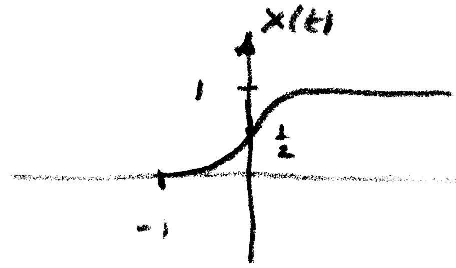
$$\frac{d}{dt}x'(t) \xleftrightarrow{\mathcal{F}} \frac{2(j\omega)}{\omega^2}[1-\cos(\omega)] = \frac{2j}{\omega}[1-\cos(\omega)]$$

$$\frac{d^2}{dt^2}x'(t) \xleftrightarrow{\mathcal{F}} \frac{2j(j\omega)}{\omega}[1-\cos(\omega)] = 2[\cos(\omega)-1]$$

which verifies the answer to part i)

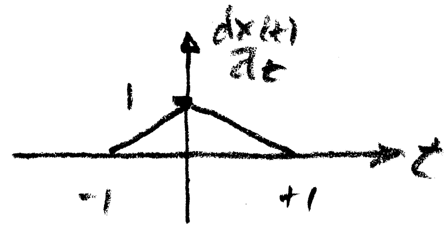
iii)

$$x(t) = \begin{cases} 0 & t<-1 \\ \frac{(t+1)^2}{2} & -1<t<0 \\ 1-\frac{(1-t)^2}{2} & 0<t<1 \\ 1 & t>+1 \end{cases}$$



Taking the derivative of $x(t)$ yields

$$\frac{dx(t)}{dt} = \begin{cases} 0 & t<-1 \\ t & -1<t<0 \\ 1-t & 0<t<1. \\ 0 & t>+1 \end{cases}$$



Thus

$$\frac{dx(t)}{dt} = x'(t)$$

$$x(t) = \int_0^t x'(\tau)\,d\tau$$

Then, if we apply the integration property of F.T.s

$$X(j\omega) = \frac{1}{j\omega} X'(j\omega) + \pi X'(0)\,\delta(\omega)$$

Now

$$X'(j\omega) = \frac{2}{\omega^2}\left[1 - \cos(\omega)\right]$$

and we also need $X'(0)$. But both numerator and denominator go to zero as $\omega \to 0$ so we need to differentiate both

	Numerator	Denominator
	$\frac{d}{d\omega}\left[1-\cos(\omega)\right] = \sin(\omega)$	$\frac{d(\omega^2)}{d\omega} = 2\omega$
	$\frac{d^2}{d\omega^2}\left[1-\cos(\omega)\right] \to \cos(\omega)$	$\frac{d^2(\omega^2)}{d\omega^2} = 2$

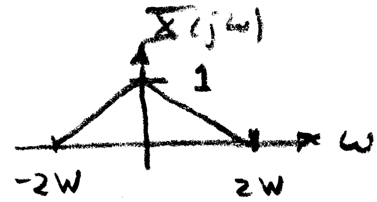
Thus $X'(0) = \dfrac{2 \lim_{\omega \to 0} (\cos \omega)}{2} = 1$

So $X(j\omega) = \frac{1}{j\omega^3}\left[1 - \cos(\omega)\right] + \pi\delta(\omega)$

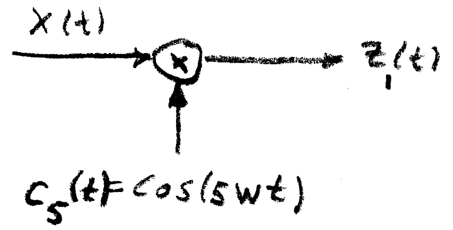
59) Problem 8.22 in the text

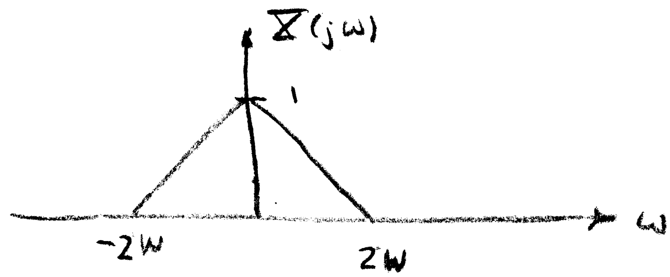
The input is

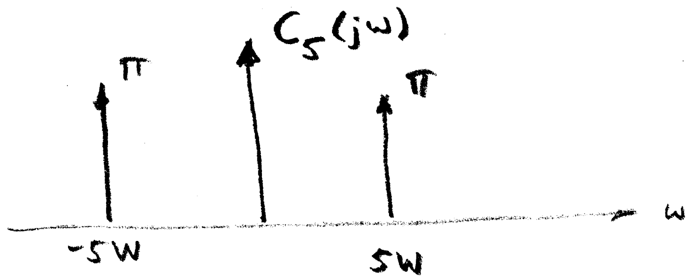
$X(j\omega) =$

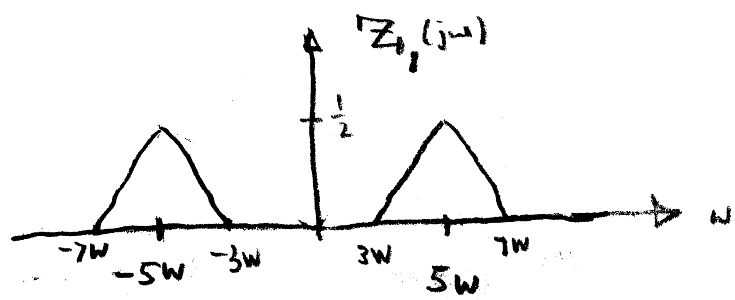


The input amplitude modulates a carrier at frequency 5W

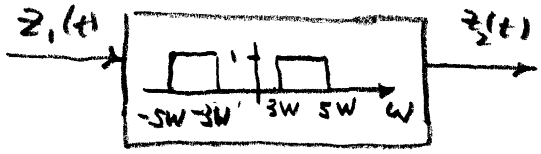


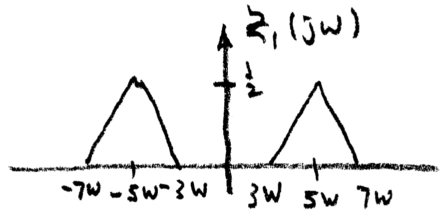


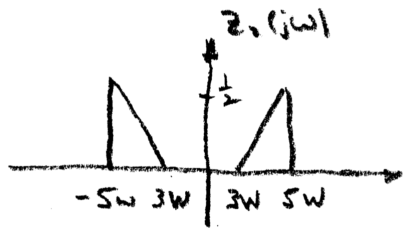




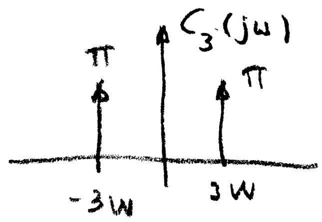
The Signal $z_1(t)$ is bandpass filtered



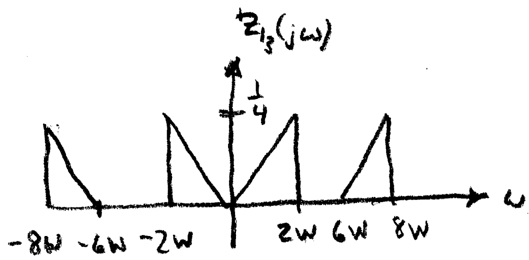




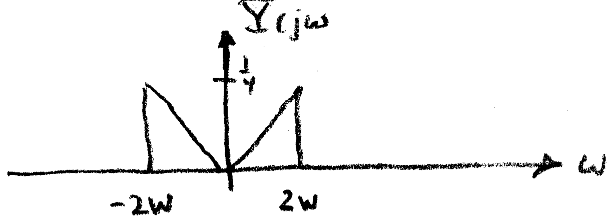
The $Z_2(t)$ signal modulates a cosine carrier at $3W$



which yields $z_3(t)$ with frequency response



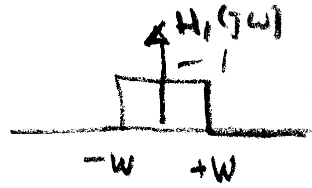
Then low pass filtering over the band $\pm 3W$ yields $Y(j\omega)$



510) The $H(j\omega)$ function is not absolutely integrable

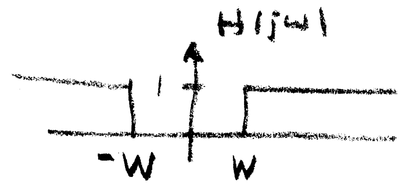
However, if we define

$$H_1(j\omega) = \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| > W \end{cases}$$



Then

$$H(j\omega) = 1 - H_1(j\omega)$$



The inverse F.T. of $H_1(j\omega)$ is

$$h_1(t) = \frac{1}{2\pi}\int_{-W}^{W} e^{j\omega t}\,d\omega = \frac{1}{2\pi}\left[\frac{e^{j\omega t}}{jt}\right]_{-W}^{W} = \frac{1}{\pi t}\left[\frac{e^{jWt} - e^{-jWt}}{2j}\right]$$

$$= \frac{\sin(Wt)}{\pi t}$$

Also, the inverse F.T. of 1 is $\delta(t)$,

so the impulse response of the high pass filter is

$$h(t) = \delta(t) - \frac{\sin(Wt)}{\pi t}$$